# Using Representations to Learn & Teach Fractions (3-5)

MaTHink 2017

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**Understanding & Teaching Fractions 3-5**
**Using Representations to Teach Fractions**

The goal of this presentation is to be able to use models to teach fractions through real-world contexts and problems. Students need to make meaning of situations involving fractions and to be able to explain and use labeled diagrams, number lines and area models to represent, solve, and justify answers. We will look at materials and activities to support understanding.

The Numbers and Operations- Fractions, 3-5 Progressions makes specific recommendations on teaching fractions. Because the foundation of fractions is built in the third grade and is the basis for later work, significant time will be spent on 3rd grade material- unit fractions on the number line, different size wholes, comparing unit fractions, and seeing unit fractions as countable quantities similar to whole numbers. We will also work with 5th grade multiplication of fractions using area models.

Use models to teach fractions in real-world contexts; explain, use diagrams, number lines and area models to represent, solve, and justify answers

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1.) We start by giving students experiences that will build an intuitive understanding of fraction unit fractions built on fair shares. This will use mostly area and linear models building on grades 1 and 2.- Create fraction bars and move to the number line. Know that the whole can change and that that alters the amount of the fraction
2.) Building on whole number understanding of counting and placing a distinct quantity on a number line

**Using Representations to Teach Fractions (Number lines just above 2)**

Use models to teach fractions in real-world contexts; explain, use diagrams, number lines and area models to represent, solve, and justify answers **(Number lines just above 2)**

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1.) We start by giving students experiences that will build and intuitive understanding of fraction unit fractions built on fair shares. This will use mostly area and linear models building on grades 1 and 2.- Know that the whole can change and that that alters the amount of the fraction. Create fraction bars and move to the number line.
   a.) Fair share - drawing, making and breaking, [not groups of objects- avoid confusion] pasta, blocks
   b.) Paper folding and comparing
   c.) Fractions in the world- on boards, windows… SLIDE?
   d.) Fair shares, Area models, may look different
   e.) Create number line from fraction bars & draw number line

Understanding Fair Shares- recognizable quantity- dependent on the whole or 1, via area models and number lines. Fractions + Estimation + regular and irregular shares

2.) Building on whole number understanding of counting and placing a distinct quantity on a number line. Look at benchmark fractions of ½ and 1.???
   a.) Draw some number lines and see how much is a whole and two wholes
   b.) Benchmark Estimate ½ and 1

3.) Equivalence
4.) Composing fractions from unit fractions
5.) Area models and number lines addition and mult.- culminates in 5th mult.models

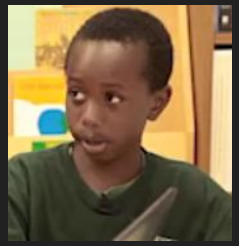
# Outcome Video: What & How did they know? PS

5min.

Cecilia $^{11}/_{12} + \frac{1}{5}$ Is it more than 1?

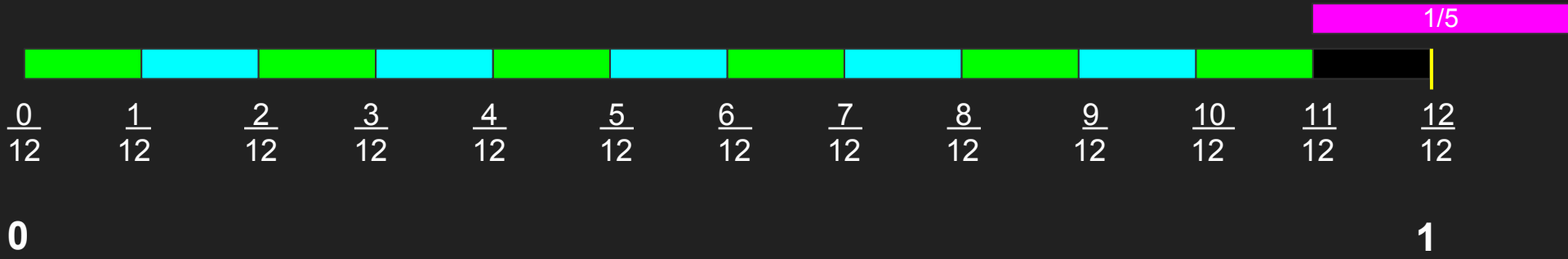
Video $\frac{5}{8}$ or $^{5}/_{12}$ Monica benchmark number line


Malcolm $\frac{3}{8}$ & $\frac{5}{6}$



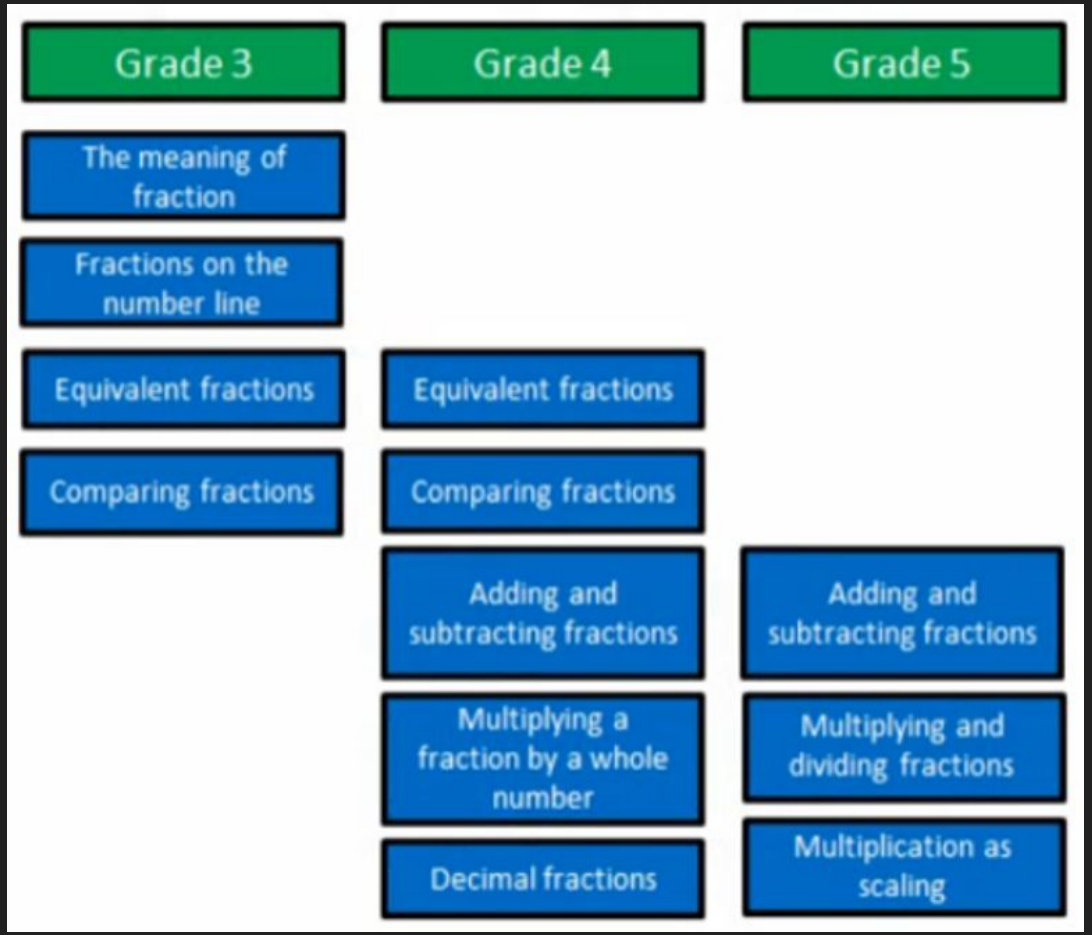
Multiple representations for $^{1}/_{2}$

# $\frac{11}{12} + \frac{1}{5}$ Is it more than one?



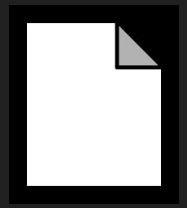
# Fraction Standards

## Representations




Illustrative Mathematics

# Fair Shares- Unit Fractions



Intuitive understanding  No symbolic representation

Grades 1 and 2  Handout

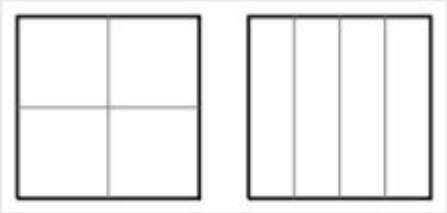
Shading Parts of a Figure  Questions- Are there the right number of shares?

-Are they fair shares or equal size

- One half, one third, one fourth
- Two thirds, two fourths
- Three fourths

# Mathematical Progressions

**Squares partitioned into fourths**



These different partitions of a square afford the opportunity for students to identify correspondences between the differently-shaped fourths (MP.1), and to explain how one of the fourths on the left can be transformed into one of the fourths on the right (MP.7).

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

**Different pattern blocks compose a regular hexagon**

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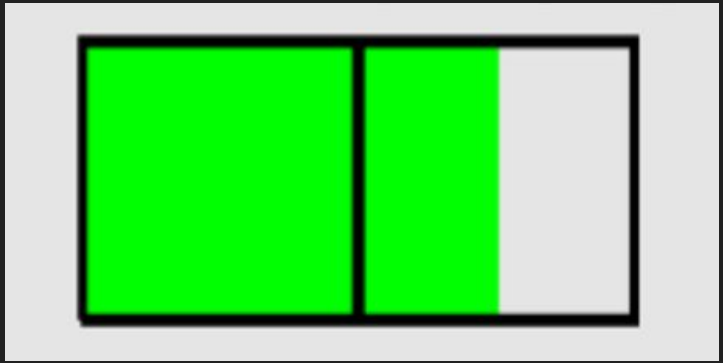
# 3rd Grade:

Area representations of $\frac{1}{4}$

*In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.*

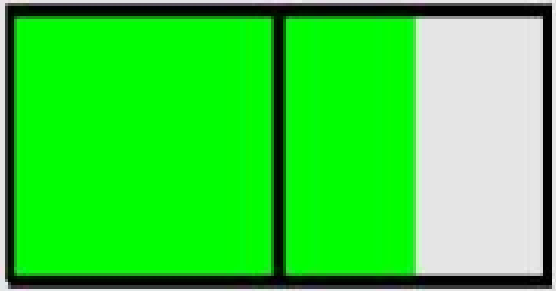
# How much is this?



# Identifying The Whole

3.NF.1

## The importance of specifying the whole

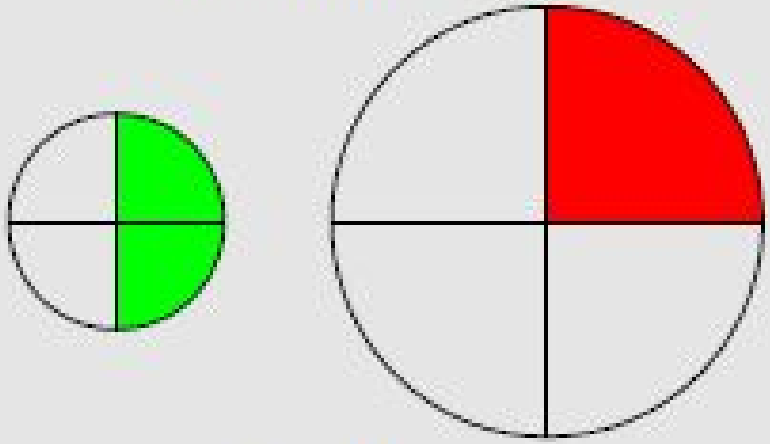


*Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.*

# The whole determines the relative size of fractions

# Identifying The Whole

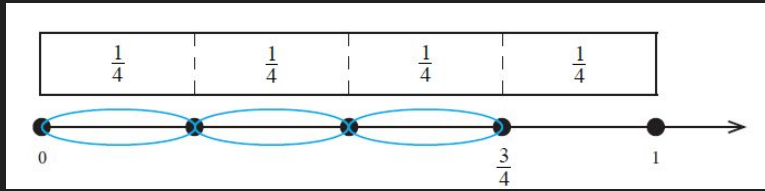
The importance of referring to the same whole when comparing fractions



*A student might think that* $\frac{1}{4} > \frac{1}{2}$*, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.*

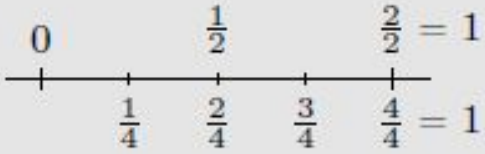
# Unit fractions -Fair Shares- Area & Number Line 20 min.

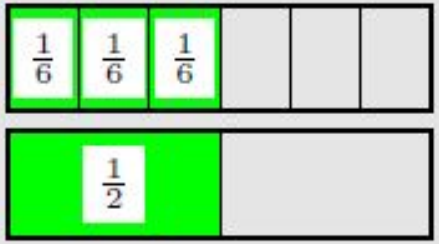
- Show & Tell- Formative Assessment for equal portions - assess for fair share understanding - Ss will explain why it is a fourth or why not
- Ex. Areas On Wall - ½ - 1/10 - Intuitive understanding
  - Expands our the unit fractions to tenths- area model for intuitive understanding
- Copy paper: Folding and labeling different ways ½, ¼. ⅛ - Explain & justify your partitioning- are the partitions equal in area, are there the right number of pieces or *fair shares*? Shows size and equivalence
- Bar model from strips ½ ⅓ ¼ (⅛) ----- on wall to form a number line, then draw



# Equivalence

Using the number line and fraction strips to see fraction equivalence




3.NF.3abc Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

Finding equivalence using bar models- fraction bars or double number lines

# Benchmarks

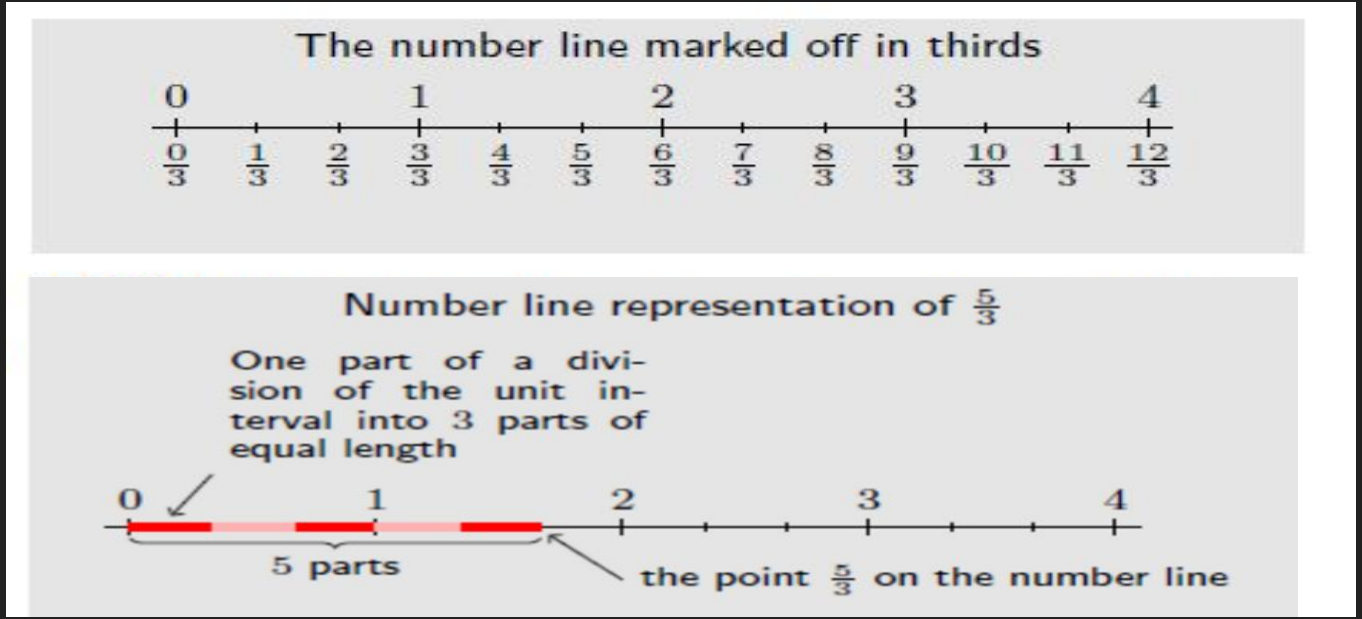
Critical Benchmarks  0, ½, 1

Student Thinking

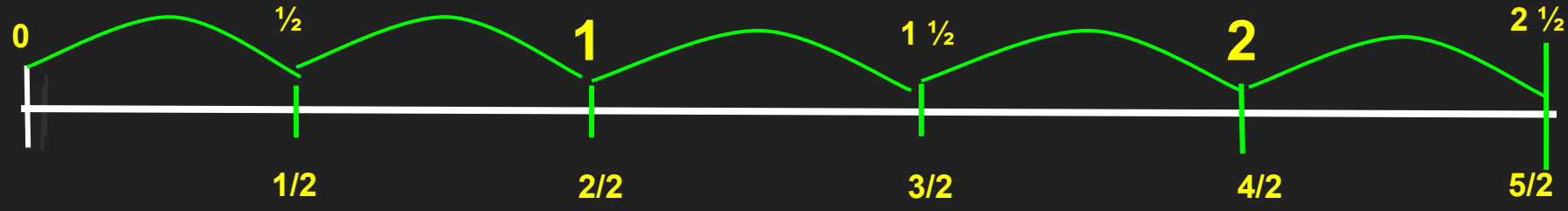
- Equivalence to ½- 2/4,  3/6…
- Seeing linear and area model
- Benchmark *Fraction* Sheet

# Unit Fractions on a Number Line - fraction as a number

Graphic Representation to symbolic notation ⅓ + ⅓… or ⅔ + ⅓ or 1



# ½+½+½+½+½ OR 5 x ½ OR 2 ½ ÷ ½




Multiplication & Division are Inverse Operations...

Repeated Addition and Subtraction

# Build these on a number line

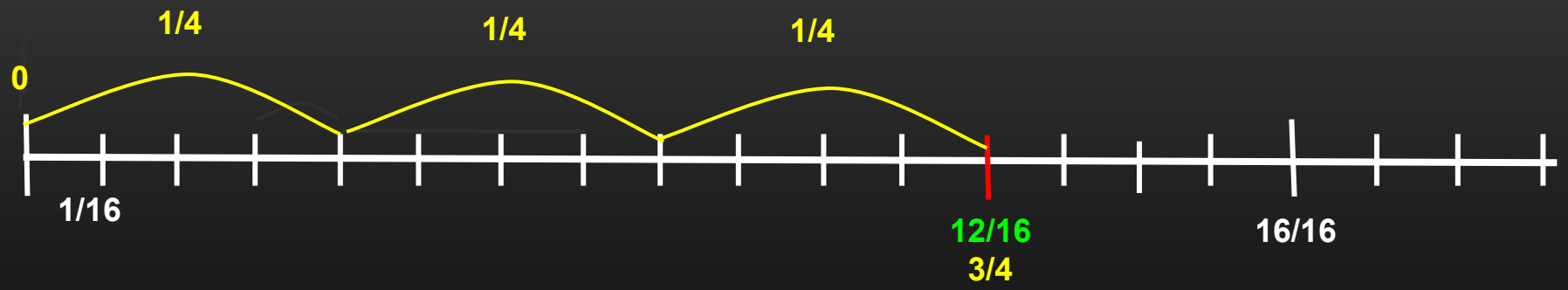
$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$

$4 \times \frac{1}{3} =$

$1\ \frac{1}{3} \div \frac{1}{3} =$

# Using Unit Fractions and Equivalence to Solve

$$\frac{12}{16} \div \frac{1}{4}$$




Or… $^{12}/_{16} = \frac{3}{4}$ and $\frac{3}{4} \div \frac{1}{4} = \frac{1}{4}$

# Adding Fractions Using Common Denominators

Fourths

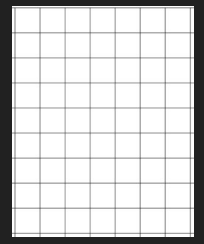
# Adding Fractions Using Common Denominators

Thirds

# Adding Fractions Using Common Denominators



# Area Models- 4th & 5th

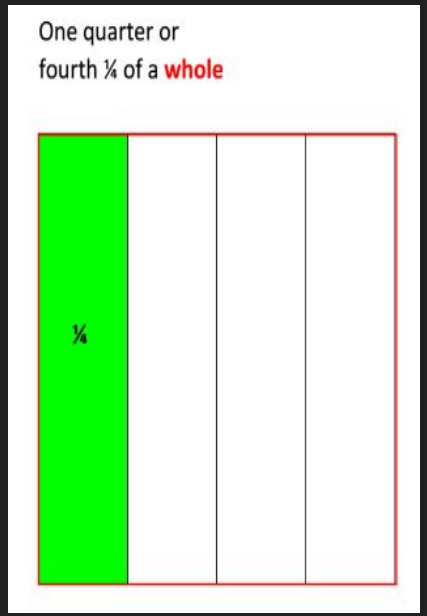


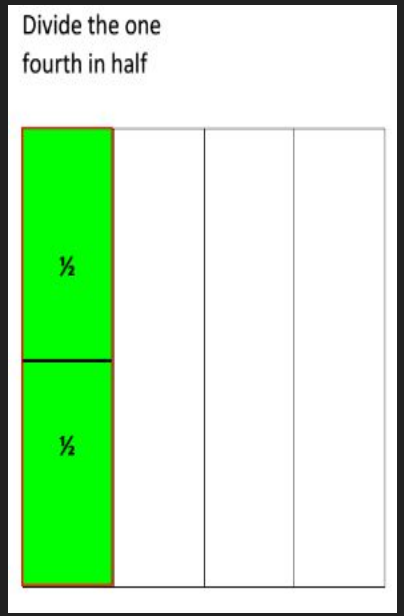
Area model similar to whole numbers for graphics equivalent fractions

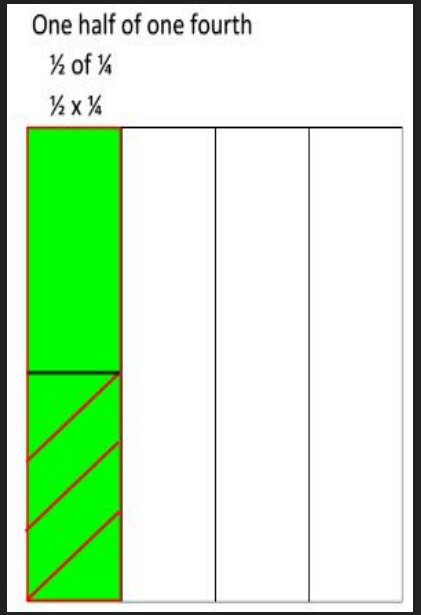
Vertical- Horizontal

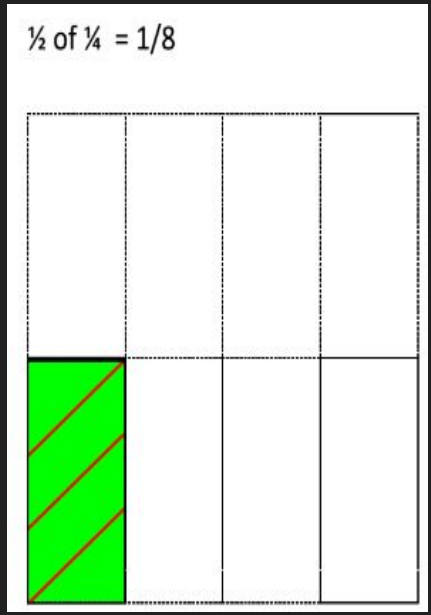
- ½ + ⅓
- ⅓ + ¼

# ½ x ¼




Try ½ x ⅓ on graph or blank paper

# References & Resources

CC Progressions Number and Operations—Fractions, 3–5

CA Mathematics Frameworks by Grade-Level

Illustrative Mathematics Fraction Progressions

Contact Steve Sher: ssher@rusd.k12.ca.us